

1. Postulates of Quantum Mechanics

❖ Definition:

- ❖ Postulates are the **basic foundational principles** upon which the entire structure of quantum mechanics is built.
They cannot be derived from other theories; instead, they are **assumptions verified by experiments** and used to describe microscopic systems such as electrons, atoms, and photons.

❖ 1.State Postulate (Wavefunction Postulate)

- ❖ The **state** of a quantum system is completely specified by a **wavefunction** $\psi(x,t)$.
- ❖ $\psi(x,t)$ contains **all measurable information** about the system.
- ❖ The probability of finding the particle like electron in a region between x and $x+dx$ is : $P(x)dx = |\psi(x,t)|^2 dx$
- ❖ The wavefunction must satisfy the **normalization condition**: $\int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = 1$
- ❖ **Example:** The wave function describes how likely it is to find an electron at a certain distance from the nucleus at a given time.

❖ 2.Observable Postulate

- ❖ Each **measurable (observable) physical quantity** (like position, momentum, energy) is represented by a **linear Hermitian operator** $A^{\hat{}}$.
- ❖ The **expectation value** (average measurement) of the observable is $\langle A \rangle = \int \psi^*(x) A^{\hat{}} \psi(x) dx$
- ❖ Hermitian property ensures **real eigenvalues**, which correspond to measurable results.
- ❖ **Example:**
 - Position(x) $\rightarrow x^{\hat{}}$
 - Momentum(p) $\rightarrow p^{\hat{}} = -i\hbar d/dx$
 - Energy(E) (Hamiltonian) $\rightarrow H^{\hat{}} = T^{\hat{}} + V^{\hat{}} = -\hbar^2/2m + V(x)$

❖ 3. Measurement Postulate

- ❖ When we measure an observable (quantity), the system will only be found in one of the operator's eigenvalues.
- ❖ This means the measurement outcome is quantized (discrete values).
Mathematically: $\hat{A}\psi = a\psi$
- ❖ **Where:** \hat{A} = operator
 - ψ = wave function
 - a = eigenvalue (measured value)
- ❖ [The probability of getting a particular eigenvalue 'ai' is given by $|c_i|^2$

Where c_i is the coefficient in $\psi = \sum i c_i \psi_i$, c_i is the coefficient]

- ❖ **Example:** For momentum measurement: $\hat{p}\psi = \hbar k \psi$
- ❖ The measured momentum can only be one of the allowed eigenvalues $\hbar k$.

❖ 4. Time Evolution Postulate

- ❖ The **time evolution** of a quantum state is determined by the **time-independent Schrödinger wave equation**
- ❖ The wave function changes with time according to the time-independent Schrödinger wave equation: $\hat{H}\psi(x,t) = i\hbar\partial\psi/\partial t$
- ❖ This wave equation describes how the wave function evolves or moves over time and The Hamiltonian \hat{H} represents the total energy operator.
- ❖ **Example:** It predicts how an electron's position probability changes with time.

❖ 5. Expectation value postulate

- ❖ The average expected value of a measurable quantity : $\langle A \rangle = \int \psi^* A \hat{A} \psi dx$
- ❖ **Example:** Average position is the average outcome if we measured position for many identical systems.

❖ 6: Identical particles & antisymmetry

- ❖ If two or more particles are *identical* (e.g. electrons), the total wave function must change sign when we swap any two particles.
➢ $\psi(1,2) = -\psi(2,1)$

- ❖ It leads to the Pauli exclusion principle which states that no two electrons can have the same set of 4 quantum numbers.
- ❖ **Example:** In atoms, electrons fill up different energy levels because of this rule.

- ❖ **7. Superposition Principle**

- ❖ If ψ_1 and ψ_2 are possible states of a system, any linear combination
- ❖ $\psi = c_1 \psi_1 + c_2 \psi_2$ is also a possible state.
- ❖ This principle leads to **interference** and **quantum coherence** phenomena.
- ❖ **Example:**
In a double-slit experiment, the final wavefunction is the **superposition** of waves from both slits.

- ❖ **8. Probability Conservation**

- ❖ The total probability of finding a particle anywhere in space is always **1**, and this remains **constant in time**: $\int |\psi(x,t)|^2 dx = 1$
- ❖ This ensures that **probability density is conserved** during evolution.

NOTE: LAST 2 POSTULATES ARE NOT COMPULSORY