

1. Postulates of Quantum Mechanics

❖ Definition:

- ❖ Postulates are the **basic foundational principles** upon which the entire structure of quantum mechanics is built.
They cannot be derived from other theories; instead, they are **assumptions verified by experiments** and used to describe microscopic systems such as electrons, atoms, and photons.

❖ 1.State Postulate (Wavefunction Postulate)

- ❖ The **state** of a quantum system is completely specified by a **wavefunction** $\psi(x,t)$.
- ❖ $\psi(x,t)$ contains **all measurable information** about the system.
- ❖ The probability of finding the particle like electron in a region between x and $x+dx$ is : $P(x)dx = |\psi(x,t)|^2 dx$
- ❖ The wavefunction must satisfy the **normalization condition**: $\int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = 1$
- ❖ **Example:** The wave function describes how likely it is to find an electron at a certain distance from the nucleus at a given time.

❖ 2.Observable Postulate

- ❖ Each **measurable (observable) physical quantity** (like position, momentum, energy) is represented by a **linear Hermitian operator** \hat{A} .
- ❖ The **expectation value** (average measurement) of the observable is $\langle A \rangle = \int \psi^*(x) \hat{A} \psi(x) dx$
- ❖ Hermitian property ensures **real eigenvalues**, which correspond to measurable results.
- ❖ **Example:**
 - Position(x) $\rightarrow \hat{x}$
 - Momentum(p) $\rightarrow \hat{p} = -i\hbar \frac{d}{dx}$
 - Energy(E) (Hamiltonian) $\rightarrow \hat{H} = \hat{T} + \hat{V} = \frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x)$

❖ 3. Measurement Postulate

- ❖ When we measure an observable (quantity), the system will only be found in one of the operator's eigenvalues.

- ❖ This means the measurement outcome is quantized (discrete values).

Mathematically: $\hat{A}\psi = a\psi$

- ❖ **Where:** \hat{A} = operator

➤ ψ = wave function

➤ a = eigenvalue (measured value)

- ❖ [The probability of getting a particular eigenvalue ' a_i ' is given by $|c_i|^2$

Where c_i is the coefficient in $\psi = \sum c_i \psi_i$, c_i is the coefficient]

- ❖ **Example:** For momentum measurement: $\hat{p}\psi = \hbar k\psi$

- ❖ The measured momentum can only be one of the allowed eigenvalues $\hbar k$.

❖ 4. Time Evolution Postulate

- ❖ The **time evolution** of a quantum state is determined by the **time-independent Schrödinger wave equation**

- ❖ The wave function changes with time according to the time-independent

Schrödinger wave equation: $\hat{H}\psi(x,t) = i\hbar \partial\psi/\partial t$

- ❖ This wave equation describes how the wave function evolves or moves over time and The Hamiltonian \hat{H} represents the total energy operator.

- ❖ **Example:** It predicts how an electron's position probability changes with time.

❖ 5. Expectation value postulate

- ❖ The average expected value of a measurable quantity : $\langle A \rangle = \int \psi^* \hat{A} \psi dx$

- ❖ **Example:** Average position is the average outcome if we measured position for many identical systems.

❖ 6: Identical particles & antisymmetry

- ❖ If two or more particles are *identical* (e.g. electrons), the total wave function must change sign when we swap any two particles.

➤ $\psi(1,2) = -\psi(2,1)$

- ❖ It leads to the Pauli exclusion principle which states that no two electrons can have the same set of 4 quantum numbers.
- ❖ **Example:** In atoms, electrons fill up different energy levels because of this rule.

❖ 7.Superposition Principle

- ❖ If ψ_1 and ψ_2 are possible states of a system, any linear combination
- ❖ $\psi = c_1 \psi_1 + c_2 \psi_2$ is also a possible state.
- ❖ This principle leads to **interference** and **quantum coherence** phenomena.
- ❖ **Example:**
In a double-slit experiment, the final wavefunction is the **superposition** of waves from both slits.

❖ 8.Probability Conservation

- ❖ The total probability of finding a particle anywhere in space is always **1**, and this remains **constant in time**: $\int |\psi(\mathbf{x},t)|^2 d\mathbf{x} = 1$
- ❖ This ensures that **probability density is conserved** during evolution.

NOTE: LAST 2 POSTULATES ARE NOT COMPULSERY