2. Operators in Quantum Mechanics

- ## Definition
- An **operator** is a mathematical rule that acts on a wavefunction ψ to give another function.
  Operators are used to extract **observable quantities** (like momentum, energy, angular momentum) from ψ.
- Mathematically: $\hat{A}\psi = a\psi$
- Example:
  - Position operator, $\hat{x}$
  - Momentum operator, $\hat{p}$
  - Energy operator (Hamiltonian), $\hat{H}$
- Each act on wave functions to produce either another function or, for eigenfunctions, scale the function by some value.

- ## Types of Operators

1.Linear operators

- An operator $\hat{O}$ is linear if: $\hat{O}(a\psi_1 + b\psi_2) = a\hat{O}\psi_1 + b\hat{O}\psi_2$
- **Example:**

  Position operator, $\hat{x}$

  Momentum operator, $\hat{p}$

  Energy operator (Hamiltonian), $\hat{H}$
- It supports the superposition principle, essential for quantum theory.
- Most common quantum operators (position, momentum, energy) are linear.

2.Hermitian (self-adjoint) operators

- $\hat{O}$ is Hermitian if: $\int \psi_1(\hat{O}\psi_2)\, dx = \int (\hat{O}\psi_1)^* \psi_2 dx$
- All the eigenvalues of Hermitian operators are real.
- The eigenfunctions of Hermitian operators form an orthonormal basis.
- All physical observables correspond to Hermitian operators.

3.Unitary Operators

- A unitary operator preserves the norm of the wave function:

$$\hat{U}^\dagger \hat{U} = \hat{U}\hat{U}^\dagger = 1$$

- The probability interpretation (norm of state vector) remains unchanged by unitary operations.
- These operators are used for time evolution & transformations in Hilbert space.
- The quantum evolution operator in the Schrödinger equation is unitary.

4. Projection Operators

- An operator $\hat{P}$ is a projection operator if: $\hat{P}^2 = \hat{P}$
- An operator that, when applied twice to a given state vector, produces the same result as applying it once is called a projection operator:
- For example, projecting a quantum state onto an eigenspace associated with a particular measurement value.

5.Fundamental Operators

- Mathematical tools that correspond to physical quantities like position, momentum, and energy.

Observable	Operator	Example/Expression
Position (x)	$\hat{x}$	Multiplication by x
Momentum (P)	$-i\hbar\, d/dx$	Differentiation with respect to x
Kinetic energy (T)	$p^2/2m$	Differentiation with respect to x
Potential energy (V)		Function $V(x)$ acting on wave function
Total energy (H)	$\hat{T}+\hat{V}$	The Hamiltonian operator

6.Algebra of Operators (Operator Manipulation)

- A system of rules for combining linear operators (which represent physical observables like position and momentum) through addition, multiplication, and other operations, governed by algebraic principles
- Addition:

  $$(\hat{A}+\hat{B})\varphi=\hat{A}\varphi+\hat{B}\varphi$$

- Multiplication:

  $$(\hat{A}\hat{B})\varphi=\hat{A}(\hat{B}\varphi)$$

- Commutator:

  $$[\hat{A},\hat{B}]=\hat{A}\hat{B}-\hat{B}\hat{A}$$