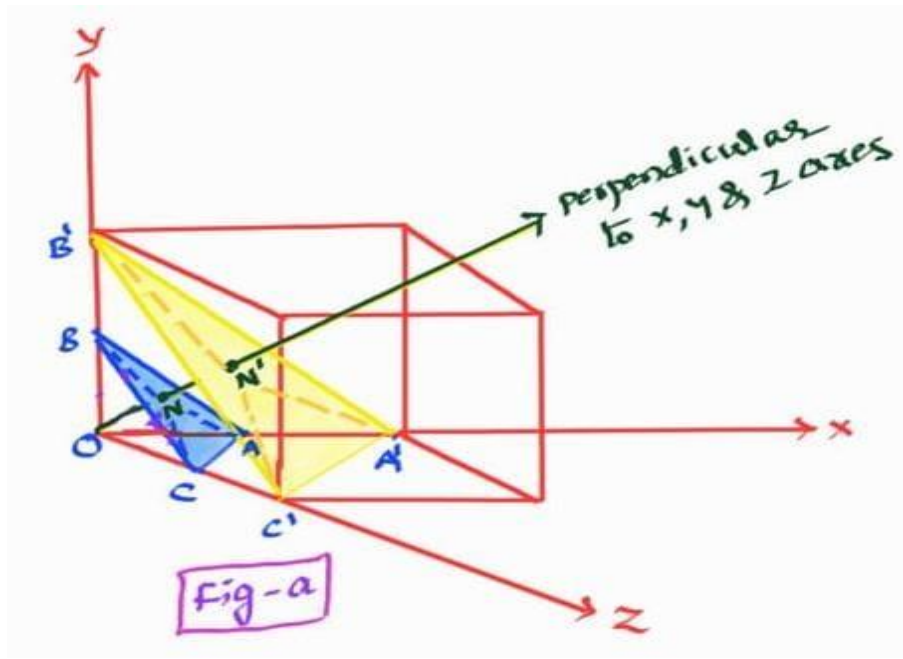


INTERPLANAR DISTANCE (OR) INTERPLANAR SPACING (OR) d-SPACING IN CUBIC LATTICE (OR) SEPARATION BETWEEN LATTICE PLANES IN A CUBIC CRYSTAL(d)

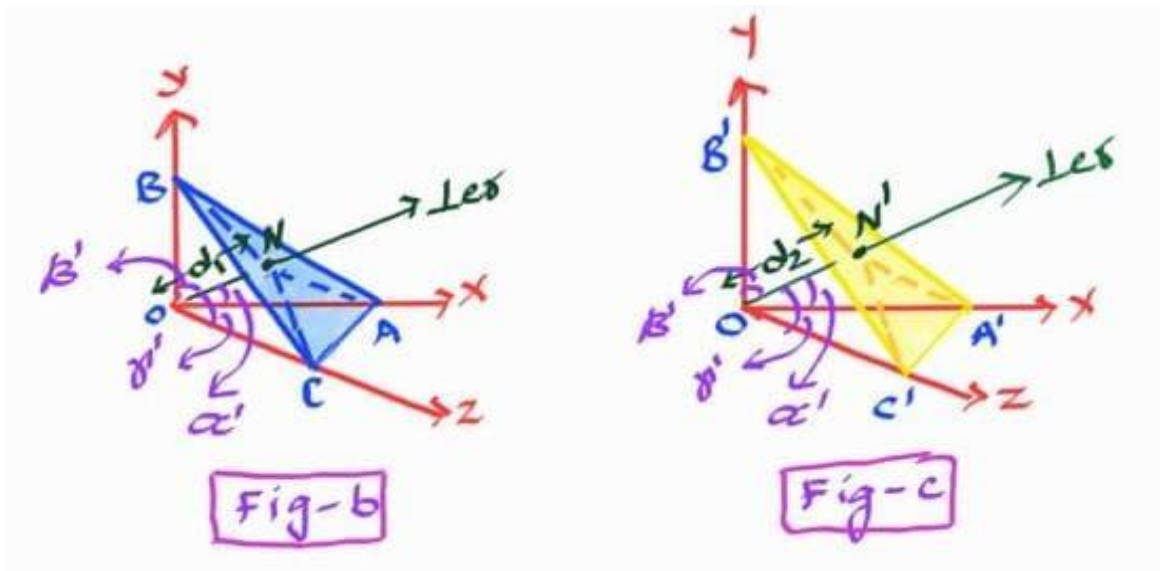
- ❖ **Definition:** The distance between any two successive parallel planes in a cubic crystal is called interplanar distance or d-spacing or separation between lattice planes.
- ❖ **Derivation/Explanation:**
- ❖ Let us consider a cubic crystal with two successive parallel planes ABC and A'B'C'
- ❖ Let (h k l) be the Miller indices of the two planes ABC and A'B'C' [Fig-a]
- ❖ Let $ON=d_1$ be the perpendicular distance of the first plane ABC from the origin and α, β, γ be the angle between ON and crystal axes x, y, z respectively.



- ❖ The intercepts of the planes on the x, y, z axes are
- ❖ $OA = a/h, OB = b/k, OC = c/l$
- ❖ since, in a cubic crystal $a=b=c$
- ❖ $OA = a/h, OB = a/k, OC = a/l$ (1)
- ❖ And $x=d_1 \cos \alpha, y=d_1 \cos \beta$ & $z=d_1 \cos \gamma$ (2)
- ❖ Also $ON^2 = x^2 + y^2 + z^2$
- ❖ From(2) : $ON^2 = d_1^2 \cos^2 \alpha + d_1^2 \cos^2 \beta + d_1^2 \cos^2 \gamma$

$$d_1^2 = d_1^2 (\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma) \text{ (since } ON = d_1)$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \text{(3)}$$



- ❖ From the figure “b”
- ❖ $\cos \alpha' = ON/OA = d_1 / OA$; $\cos \beta' = ON/OB = d_1 / OB$; $\cos \gamma' = ON/OC = d_1 / OC$ (4)
- ❖ From (3)&(4) : $(d_1/OA)^2 + (d_1/OB)^2 + (d_1/OC)^2 = 1$
- ❖ From (1): $[d_1/(a/h)]^2 + [d_1/(a/k)]^2 + [d_1/(a/l)]^2 = 1$

$$(d_1^2 h^2 / a^2) + (d_1^2 k^2 / a^2) + (d_1^2 l^2 / a^2) = 1$$

$$(d_1^2 / a^2) [h^2 + k^2 + l^2] = 1$$

$$d_1^2 (h^2 + k^2 + l^2) = a^2$$

$$d_1^2 = a^2 / (h^2 + k^2 + l^2)$$

$$d_1 = a / \sqrt{(h^2 + k^2 + l^2)} \text{(5)}$$

- ❖ Let $ON' = d_2$ be the perpendicular distance of the second plane $A'B'C'$ parallel to the first plane ABC
- ❖ The intercepts of this plane on the x, y, z axes are
- ❖ $OA' = 2a/h$, $OB' = 2a/k$, $OC' = 2a/l$ (6)
- ❖ And $x = d_2 \cos \alpha$, $y = d_2 \cos \beta$ & $z = d_2 \cos \gamma$ (7)
- ❖ Also $(ON')^2 = x^2 + y^2 + z^2$
- ❖ From (7) : $(ON')^2 = d_2^2 \cos^2 \alpha' + d_2^2 \cos^2 \beta' + d_2^2 \cos^2 \gamma'$

$$d_2^2 = d_2^2 [\cos^2 \alpha' + \cos^2 \beta' + \cos^2 \gamma'] \text{ (since } ON' = d_2)$$

$$\cos^2 \alpha' + \cos^2 \beta' + \cos^2 \gamma' = 1 \text{ (8)}$$

- ❖ From the figure (c):
- ❖ $\cos \alpha' = ON'/OA' = d_2/OA'$; $\cos \beta' = ON'/OB' = d_2/OB'$; $\cos \gamma' = ON'/OC' = d_2/OC'$ (9)

- ❖ From (8) & (9): $(d_2/OA')^2 + (d_2/OB')^2 + (d_2/OC')^2 = 1$

- ❖ From (6) $[d_2/(2a/h)]^2 + [d_2/(2a/k)]^2 + [d_2/(2a/l)]^2 = 1$

$$d_2^2 h^2 / 4a^2 + d_2^2 k^2 / 4a^2 + d_2^2 l^2 / 4a^2 = 1$$

$$(d_2^2 / 4a^2) [h^2 + k^2 + l^2] = 1$$

$$d_2^2 = 4a^2 / (h^2 + k^2 + l^2)$$

$$d_2 = (2a) / \sqrt{(h^2 + k^2 + l^2)} \text{(10)}$$

- ❖ Thus, Inter planar distance between two adjacent parallel planes of miller indices (h k l) in a cubic crystal (lattice) is given by

$$d = d_2 - d_1$$

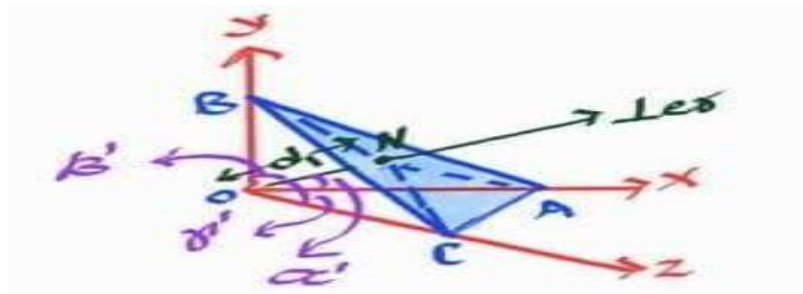
$$= (2a) / \sqrt{(h^2 + k^2 + l^2)} - a / \sqrt{(h^2 + k^2 + l^2)}$$

$$d = a / \sqrt{(h^2 + k^2 + l^2)} \text{(11)}$$

ALTERNATE METHOD

❖ Derivation/Explanation:

- ❖ Let us consider a cubic crystal with plane ABC and the next plane parallel to the ABC passes through the origin(O).
- ❖ Let $ON=d$ be the perpendicular distance of the plane ABC from the origin(O) and α', β', γ' be the angle between ON and crystal axes x, y, z respectively.



- ❖ Let (h k l) be the Miller indices of the plane ABC Then intercepts of the plane on the x, y, z axes are

$$OA = a/h, \quad OB = b/k, \quad OC = c/l$$

- ❖ **since**, in a cubic crystal $a=b=c$
- ❖ **$OA = a/h, \quad OB = a/k, \quad OC = a/l$ (1)**
- ❖ And **$x=dcos\alpha, \quad y=dcos\beta \text{ \& } z=dcos\gamma$ (2)**
- ❖ Also $ON^2 = x^2 + y^2 + z^2$
- ❖ From(2) : $ON^2 = d^2\cos^2\alpha + d^2\cos^2\beta + d^2\cos^2\gamma$

$$d^2 = d^2 (\cos^2\alpha + \cos^2\beta + \cos^2\gamma) \quad (\text{since } ON=d)$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad \text{.....(3)}$$

- ❖ From the figure “b”
- ❖ **$\cos\alpha' = ON/OA=d/OA; \quad \cos\beta' = ON/OB=d/OB; \quad \cos\gamma'=ON/OC =d/ OC$ (4)**
- ❖ Form (3)&(4) : $(d/OA)^2 + (d/OB)^2 + (d/OC)^2 = 1$
- ❖ From (1): $[d/(a/h)]^2 + [d/(a/k)]^2 + [d/(a/l)]^2 = 1$

$$(d^2 h^2 / a^2) + (d^2 k^2 / a^2) + (d^2 l^2 / a^2) = 1$$

$$(d^2 / a^2) [h^2 + k^2 + l^2] = 1$$

$$d^2 (h^2 + k^2 + l^2) = a^2$$

$$d^2 = a^2 / (h^2 + k^2 + l^2)$$

$$\boxed{d = a / \sqrt{(h^2 + k^2 + l^2)} \quad \text{.....(5)}}$$