8.Expectation Value in Quantum Mechanics

❖ The concept of the **expectation value** is central to quantum mechanics, as it connects the mathematical description of a quantum system (the wave function and operators) to the measurable, observable properties of the system.

❖ Definition:

❖ The expectation value of a dynamical quantity is the **mathematical average** for the result of a single measurement performed on a system.

❖ In other words, if one were to perform many measurements of a physical quantity (like position, momentum, or energy) on a system described by the wave function ψ, the expectation value represents the average of all those measurements.

❖ In quantum mechanics, every measurable physical quantity such as position, momentum, or energy is represented by a linear Hermitian operator acting on the system's wave function $\psi(x,t)$

The expectation value of an operator $\hat{A}$ gives the average value of the observable quantity it represents: $\langle \hat{A} \rangle = \int \psi^*(x,t)\, \hat{A}\, \psi(x,t)\, dx$

❖ **Where:**

- ❖ $\langle \hat{A} \rangle$: Expectation value (average measurable value) of the operator $\hat{A}$.
- ❖ $\psi(x,t)$: Wave function representing the state of the quantum system.
- ❖ $\psi^*(x,t)$: Complex conjugate of the wave function.
- ❖ dx : Integration over all relevant degrees of freedom (for a multi-particle system, it spans all spatial coordinates).

❖ If $\psi(x,t)$ is normalized, i.e., $\int \psi^*(x,t)\psi(x,t)\, dx = 1$

❖ then the expectation value formula directly yields a proper average.

❖ **Example: Expectation Value for an Eigenfunction**

❖ If the wave function ψ is an eigenfunction of the operator $\hat{A}$ with eigenvalue a, such that $\hat{A}\psi = a\psi$,, and the wave function is normalized, then the expectation value is **Simply:** $\langle \hat{A} \rangle = a$

- This means that when a quantum system is in an eigen state of an observable, a measurement will always return the corresponding eigenvalue, making the average (expectation value) exactly that eigenvalue.

Mathematical Examples:

- **1.Expectation Value of Momentum**
- For momentum along the x-axis, the operator is $\hat{P}(x) = -i\hbar \partial/\partial x$
- **Hence,** $\langle \hat{P} x \rangle = \int \psi^*(x,t)(-i\hbar \partial/\partial x)\, \psi(x,t)\, dx$
- Simplified form: $\langle \hat{P} x \rangle = -i\hbar \int \psi^*(x,t) \partial\psi(x,t)/\partial x$
- **2.Expectation Value of Energy**
- The Hamiltonian operator $\hat{H}$ represents the total energy of the system and appears in the Schrödinger equation: $i\hbar \partial\psi/\partial t = \hat{H}\psi$
- The expectation value of energy is given by: $\langle E \rangle = \int \psi^*(x,t)\, \hat{H}\, \psi(x,t)\, dx = \int \psi^*(x,t)\, (i\hbar \partial/\partial t)\, \psi(x,t)$