7.EIGEN VALU- EIGEN FUNCTION – EIGEN EQUATION

➢ a) Eigen value

- An **eigenvalue** is a special value associated with an **operator** that acts on a function (or vector) and gives back the same function multiplied by a constant. That constant is called the **eigenvalue**.
- **Mathematically:** $\hat{A}\psi(x)=a\,\psi(x)$
- **Here:**
  - $\hat{A}$ → Operator
  - Ψ(x)→ Function (called Eigenfunction)
  - a → Constant (called Eigenvalue)
- When the operator $\hat{A}$ acts on ψ, the function doesn't change its form, only its magnitude changes by a factor 'a'.
- **Example:**
- Let the operator $\hat{\Theta} = d^2/dx$ and $\phi(x) = e^{-2x}$ then

$$\hat{\Theta}\,\phi(x) = d^2/dx\,(e^{-2x})$$

$$\hat{\Theta}\,\phi(x) = 4\,e^{-2x}$$

$$\hat{\Theta}\,\phi(x) = 4\,\phi(x)$$

- **Here:**
  - $\hat{\Theta}$ → Operator

- $\phi(x) \rightarrow$ Eigenfunction
- $4 \rightarrow$ Constant called Eigenvalue.

Physical Significance of eigen values:

- The eigenvalue, denoted as a, represents the measured value, or possible outcome, when an operator acts on its eigen function.
- Only certain discrete values are allowed as results of measurement in quantum mechanics, making this concept fundamental for the quantization of observables.
- Eigenvalues for simple operators and functions can be found analytically, but more complicated systems may require numerical or approximate methods.
- The collection of all eigenvalues of an operator is known as its spectrum.

b) Eigenfunction

Definition:

- An **eigenfunction** is a special type of function that, when operated upon by an operator, reproduces itself multiplied by a constant (the eigenvalue).
- It represents a *state* of the system that remains unchanged (in shape) under the action of a given operator — only scaled by a factor.

Mathematically: $\hat{A}\psi(x)=a\,\psi(x)$

- **Here:**
  - $\hat{A} \rightarrow$ Operator
  - $\Psi(x) \rightarrow$ Eigenfunction
  - $a \rightarrow$ Constant (called Eigenvalue)

- When the operator $\hat{A}$ acts (operated) on ψ, the function doesn't change its form, only its magnitude changes by a factor 'a'.

➢ Requirements for an Eigen Function:

- To qualify as an eigen function $\phi(x)$, the function must fulfill the following requirements:
  - $\Psi(x)$ must be single valued everywhere.
  - $\Psi(x)$ must be square integrable, meaning the integral of its modulus squared must be finite.
  - $\Psi(x)$ must be continuous everywhere and possess a continuous derivative.
  - $\Psi(x)$ should remain finite and vanish as $x \to \pm\infty$.

➢c) Eigen Equation

➢ Definition:

- The **eigen equation** (or eigenvalue equation) is the mathematical relationship that connects an operator, its eigenfunction, and eigenvalue.

➢ Mathematically: $\hat{A}\psi(x)=a\,\psi(x)$

- **Here:**
  - $\hat{A} \rightarrow$ Operator
  - $\Psi(x) \rightarrow$ Eigenfunction
  - $a \rightarrow$ Constant (called Eigenvalue)