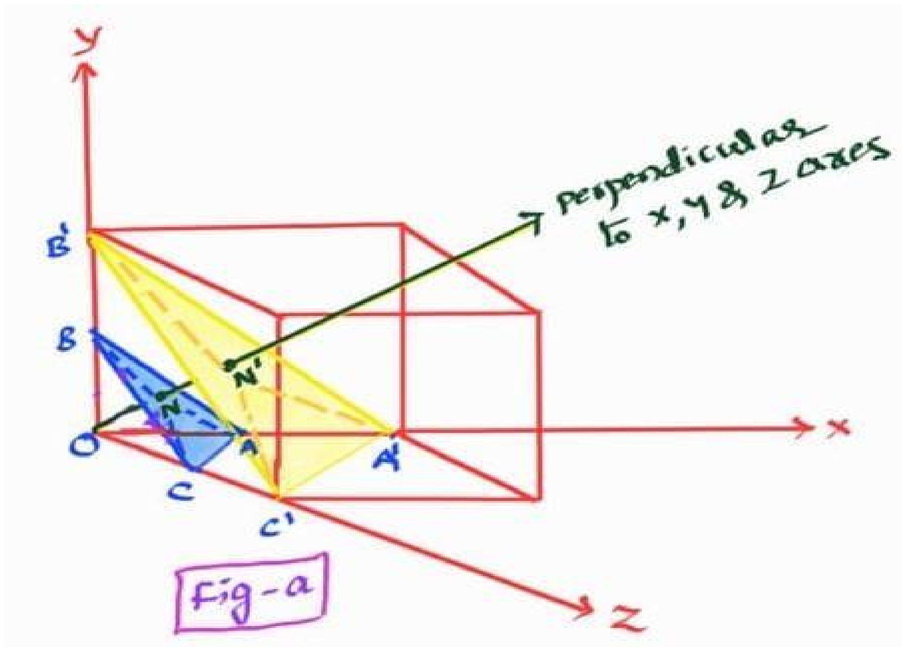
INTERPLANAR DISTANCE (OR) INTERPLANAR SPACING (OR) d-SPACING IN CUBIC LATTICE (OR) SEPARATION BETWEEN LATTICE PLANES IN A CUBIC CRYSTAL(d)

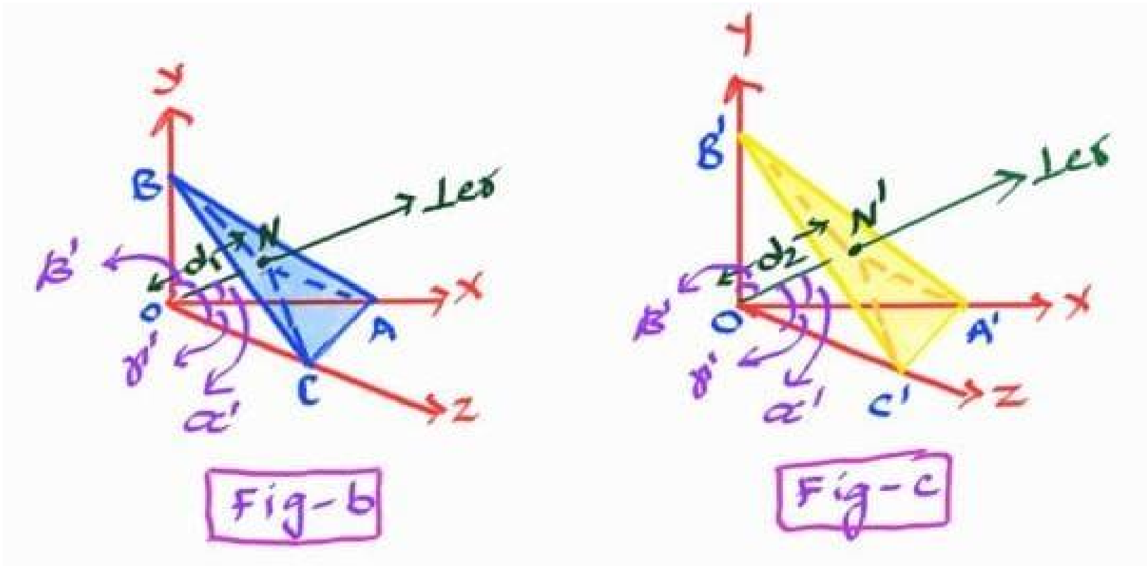
- **Definition:** The distance between any two successive parallel planes in a cubic crystal is called interplanar distance or d-spacing or separation between lattice planes.
- **Derivation/Explanation:**
- Let us consider a cubic crystal with two successive parallel planes ABC and A'B'C'
- Let (h k l) be the Miller indices of the two planes ABC and A'B'C' [Fig-a]
- Let **ON=d_1** be the perpendicular distance of the first plane ABC from the origin and α',β',γ' be the angle between ON and crystal axes x, y, z respectively.



- The intercepts of the planes on the x, y, z axes are
- OA = a/h,   OB = b/k,   OC = c/l
- since, in a cubic crystal a=b=c
- **OA = a/h,   OB = a/k,   OC = a/l ....(1)**
- And **$x=d_1\cos\alpha$, $y=d_1\cos\beta$ & $z=d_1\cos\gamma$ ....(2)**
- Also $ON^2 = x^2 + y^2 + z^2$
- From(2) : $ON^2 = d_1^2\cos^2\alpha + d_1^2\cos^2\beta + d_1^2\cos^2\gamma$

$$d_1^2 = d_1^2 (\cos^2\alpha + \cos^2\beta + \cos^2\gamma) \quad \text{(since } ON= d_1\text{)}$$

$$\mathbf{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad ....(3)}$$



- From the figure "b"
- **$\cos \alpha' = ON/OA = d_1 / OA$; $\cos \beta' = ON/OA = d_1 / OB$; $\cos \gamma' = ON/OA = d_1/ OC$ …..(4)**
- Form (3)&(4) : $(d_1/OA)^2 + (d_1/OB)^2 + (d_1/OC)^2 = 1$
- From (1): $[d_1/(a/h)]^2 + [d_1/(a/k)]^2 + [d_1/(a/l)]^2 = 1$

$$(d_1^2 h^2/ a^2) + (d_1^2 k^2/ a^2) + (d_1^2 l^2/ a^2) = 1$$

$$(d_1^2 / a^2) [h^2 + k^2 + l^2] = 1$$

$$d_1^2 (h^2 + k^2 + l^2) = a^2$$

$$d_1^2 = a^2/(h^2 + k^2 + l^2)$$

$$\mathbf{d_1 = a / \sqrt{(h^2 + k^2 + l^2)}} \quad \text{………………….(5)}$$

- Let **ON' = d_2** be the perpendicular distance of the second plane A'B'C' parallel to the first plane ABC
- The intercepts of this plane on the x, y, z axes are
- **OA' = 2a/h, OB' = 2a/k, OC' = 2a/l ………….. (6)**
- **And $x=d_2\cos\alpha$, $y=d_2\cos \beta$ & $z=d_2\cos\gamma$ ………..(7)**
- Also $(ON')^2 = x^2 + y^2 + z^2$
- From (7) : $(ON')^2 = d_2^2 \cos^2 \alpha' + d_2^2 \cos^2 \beta' + d_2^2 \cos^2 \gamma'$

$$d_2^2 = d_2^2 [\cos^2 \alpha' + \cos^2 \beta' + \cos^2 \gamma'] \quad \text{(since } ON'= d_2)$$

$$\mathbf{\cos^2 \alpha' + \cos^2 \beta' + \cos^2 \gamma' = 1} \quad \text{……………….. (8)}$$

- From the figure (c):
- **$\cos\alpha' = ON'/OA' = d_2/OA'$; $\cos\beta' = ON'/OA' = d_2/OB'$; $\cos\gamma' = ON'/OA' = d_2/OC'$ ………(9)**
- From (8) & (9): $(d_2/OA')^2 + (d_2/OB')^2 + (d_2/OC')^2 = 1$
- From (6) $[d_2/(2a/h)]^2 + [d_2/(2a/k)]^2 + [d_2/(2a/l)]^2 = 1$

$$d_2^2 h^2 / 4a^2 + d_2^2 k^2 / 4a^2 + d_2^2 l^2 / 4a^2 = 1$$

$$(d_2^2 / 4a^2)\,[h^2 + k^2 + l^2] = 1$$

$$d_2^2 = 4a^2 / (h^2 + k^2 + l^2)$$

$$\mathbf{d_2 = (2a) / \sqrt{(h^2 + k^2 + l^2)}\ \ \ldots\ldots\ldots\ldots\ldots\ldots(10)}$$

- Thus, Inter planar distance between two adjacent parallel planes of miller indices (h k l) in a cubic crystal (lattice) is given by

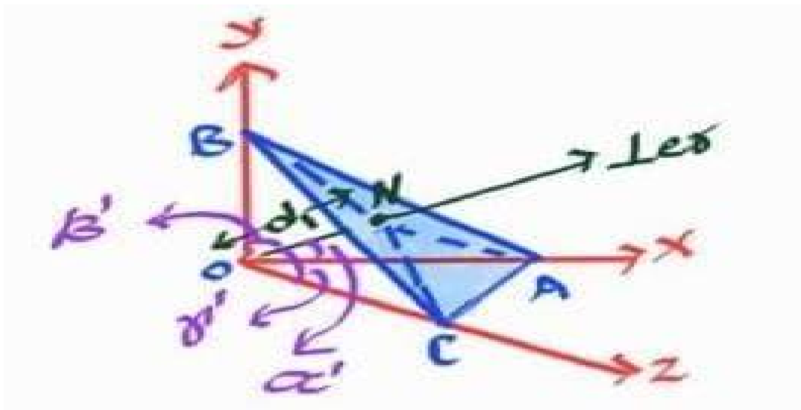
$$d = d_2 - d_1$$

$$= (2a) / \sqrt{(h^2 + k^2 + l^2)} - a / \sqrt{(h^2 + k^2 + l^2)}$$

- $$d = \mathbf{a / \sqrt{(h^2 + k^2 + l^2)}\ \ \ldots\ldots\ldots\ldots(11)}$$

ALTERNATE METHOD

- **Derivation/Explanation:**
- Let us consider a cubic crystal with plane ABC and the next plane parallel to the ABC passes through the origin(O).
- Let **ON=d** be the perpendicular distance of the plane ABC from the **origin(O)** and α',β',γ' be the angle between ON and crystal axes x, y, z respectively.



- Let (h k l) be the Miller indices of the plane ABC Then intercepts of the plane on the x, y, z axes are

  OA = a/h, OB = b/k, OC = c/l

- **since,** in a cubic crystal a=b=c
- **OA = a/h, OB = a/k, OC = a/l ………….(1)**
- And **$x = d\cos\alpha$, $y = d\cos\beta$ & $z = d\cos\gamma$ ……..(2)**
- Also $ON^2 = x^2 + y^2 + z^2$
- From(2) : $ON^2 = d^2\cos^2\alpha + d^2\cos^2\beta + d^2\cos^2\gamma$
- $\cancel{d^2} = \cancel{d^2}(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$ (since ON=d)
- **$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ …………….(3)**
- From the figure "b"
- **$\cos\alpha' = ON/OA = d/OA$; $\cos\beta' = ON/OA = d/OB$; $\cos\gamma' = ON/OA = d/OC$ ……….(4)**
- Form (3)&(4) : $(d/OA)^2 + (d/OB)^2 + (d/OC)^2 = 1$
- From (1): $[d/(a/h)]^2 + [d/(a/k)]^2 + [d/(a/l)]^2 = 1$

$$(d^2 h^2/a^2) + (d^2 k^2/a^2) + (d^2 l^2/a^2) = 1$$

$$(d^2/a^2)[h^2 + k^2 + l^2] = 1$$

$$d^2(h^2 + k^2 + l^2) = a^2$$

$$d^2 = a^2/(h^2 + k^2 + l^2)$$

$d = a/\sqrt{(h^2 + k^2 + l^2)}$ …………………(5)