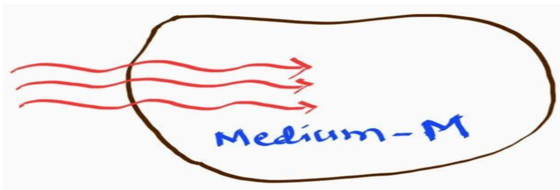
EINSTEIN COEFFICIENTS & THEIR RELATIONS

Introduction:

- Einstein Coefficients are mathematical quantities that describe the probability of a photon being absorbed or emitted by an electron (or atom or molecule).
- They are named after Albert Einstein, who proposed them in 1916.

Interaction of Light (Radiation) with Matter:



- Let us consider that as light passes through the medium **M,** three quantum processes will occur, namely Induced Absorption, Spontaneous Emission and Stimulated Emission.
- Let **N_1 and N_2** be the no. of electrons per unit volume, with the energies **E_1** and **E_2** respectively, in ground and excited states. And **E_ν** be the energy density of incident photons at frequency ν.

Induced Absorption:

- During this absorption, electrons in the ground state are raised to the excited state due to the absorption of an incident photon by an atom.
- The rate of induced absorption depends on the number of electrons in the ground state (**N_1**) and the energy density of the incident photon (**E_ν**).
- Rate of induced absorption $\propto N_1$ and Rate of induced absorption $\propto E_\nu$
- Hence Rate of induced absorption $\propto N_1E_\nu$
- Therefore, the Rate of induced absorption = $B_{12}N_1E_\nu$ ----------------- (1)
- **where:** B_{12} is the Einstein coefficient of induced absorption constant.

Spontaneous Emission:

- During this emission, an electron in the excited state returns to its ground state by emitting a photon of energy E**,** without any external agency.
- Therefore the rate of spontaneous emission depends only on the number of electrons in the excited state (N_2).
- Rate of spontaneous emission $\propto N_2$

- Therefore, the Rate of spontaneous emission = $A_{21}N_2$ ----------------- (2)
- **where:** A_{21} = Einstein coefficient of spontaneous emission constant

Stimulated Emission:

- During this emission, all the electrons in the excited state are transferred to the ground state and emit photon energy by the interaction of radiation or photon or light.
- Therefore, the rate of stimulated emission depends on the number of electrons in the metastable state (N_2) and also on the energy density of the incident photon.
- Rate of stimulated emission $\propto N_2 E\nu$
- Therefore, the Rate of stimulated emission = $B_{21} N_2 E\nu$ -------------------- (3)
- **where:** B_{21} = Einstein coefficient of stimulated emission constant

At Thermal Equilibrium:

- Rate of induced absorption = Rate of spontaneous emission + Rate of stimulated emission .
- **Equations:**

$$B_{12} N_1 E\nu = A_{21} N_2 + B_{21} N_2 E\nu$$

$$B_{12} N_1 E\nu - B_{21} N_2 E\nu = A_{21} N_2$$

$$[B_{12} N_1 - B_{21} N_2]E\nu = A_{21} N_2$$

$$E\nu = A_{21}N_2 /(B_{12}N_1 - B_{21}N_2)$$

$$= (A_{21}N_2/ B_{21}N_2)/[B_{12}N_1/ B_{21}N_2- 1]$$

$$E\nu = (A_{21} / B_{21})/[B_{12}N_1/ B_{21}N_2- 1]$$

OR

$$E\nu =(A_{21} / B_{21})/[(B_{12}/B_{21}) (N_1/ N_2)- 1] \quad \text{------------------- (4)}$$

From Boltzmann's Law: $N = N_o e^{E/kT}$

$N_1 = N_o e^{E1/kT}$ and $N2 = No\ e^{E2/kT}$

$N_1 / N_2 = N_o\, e^{E1/kT} / No\, e^{E2/kT}$

$= e^{E1/kT} / e^{E2/kT}$

$= e^{E1/kT - E2/kT} = e^{(E1-E2)/kT}$

$N_1 / N_2 = e^{h\nu/kT}$ ----------------- (5) (since $E_1 - E_2 = h\nu$)

This equation is as called Boltzmann factor

From (4) & (5)

$E\nu = (A_{21} / B_{21}) / [(B_{12} / B_{21}) e^{h\nu/kT} - 1]$

$E\nu = (A_{21} / B_{21}) \{1/[(B_{12} / B_{21})(e^{h\nu/kT} - 1)]\}$ ---------------- (6)

From Planck's radiation law, the energy density

$E\nu = 8\pi h\nu^3 / C^3 [1 / 1. (e^{h\nu/kT} - 1)]$ --------------- (7)

Comparison of Equations (6) & (7)

$A_{21} / B_{21} = 8\pi h\nu^3 / C^3 \Rightarrow A_{21} / B_{21} \propto \nu^3$ -------------------- (8)

$B_{12} / B_{21} = 1 \Rightarrow B_{12} = B_{21}$ ---------------------- (9)

Conclusions:

- **From Equation (8):** The ratio of coefficients of spontaneous to stimulated emission is proportional to the third power of the frequency of radiation.
- **From Equation (9):** When an electron with two energy levels is placed in a radiation field, the probability of an upward transition (absorption) is equal to the probability of a downward transition (stimulated emission).

Observations:

- The probability of spontaneous emission rapidly increases with frequency.
- This makes it difficult to achieve laser action in higher frequency ranges, such as X-rays.