1.8.TIME INDEPENDENT SCHRODINGER WAVE EQUATION

Introduction:

In 1926, Schrodinger described the wave nature of particles in mathematical forms, and they are known as Schrodinger's wave equations.

- **They are** 1. Time Independent Schrodinger Wave Equation (TISWE)
  2. Time Dependent Schrodinger Wave Equation (TDSWE)

Derivation of Time independent Schrodinger wave equation:

- Let us consider a particle of mass 'm' moving with a velocity 'v', which is associated with a group of waves.
- Let us suppose 'ψ' be the wave function of the particle along x, y, & z axes at a time 't' then the classical differential equation of a wave motion is given by

$$\partial^2\psi/\partial t^2 = v^2\,(\partial^2\psi/\partial x^2 + \partial^2\psi/\partial y^2 + \partial^2\psi/\partial z^2) = v^2(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2)\psi$$

$$\partial^2\psi/\partial t^2 = v^2\nabla^2\psi \text{.----------------(1)}$$

→ where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$

- The solution for the above equation is

$$\psi = \psi_0 e^{-i\omega t} \text{ --------------- (2)}$$

→where ψ_0 is the amplitude of wave function at the point (x, y, z).

- Differentiating (2) twice w.r.to 't'

$$\partial\psi/\partial t = \psi_0 e^{-i\omega t} \times (-i\omega)$$

$$\partial^2\psi/\partial t^2 = \psi_0 e^{-i\omega t} \times (-i\omega)(-i\omega)$$

$$\partial^2\psi/\partial t^2 = i^2\omega^2\psi$$

$$\partial^2\psi/\partial t^2 = -\omega^2\psi \text{ ------------ (3)}$$

- **From (1) & (3)** $-\omega^2\psi = v^2\nabla^2\psi$

$$\nabla^2\psi + \omega^2/v^2\,\psi = 0 \text{ --- -----------(4)}$$

- **But** $\omega = 2\pi\nu = 2\pi v/\lambda$ $(\because \nu = v/\lambda)$

$$\omega/\nu = 2\pi/\lambda \;\&\; \omega^2/v^2 = 4\pi^2/\lambda^2$$

- **From (4)** => $\nabla^2\psi + 4\pi^2\psi/\lambda^2 = 0$

  $\nabla^2\psi + 4\pi^2/(h^2/mv^2)\ \psi = 0$ ($\because$ $\lambda = h/P = h/mv$)

  $\nabla^2\psi + 4\pi^2 m^2 v^2 \psi/h^2 = 0$ ------------ (5)

- **We have** E = P.E + K.E

  $E = V + \frac{1}{2}mv^2$ ($\because$ P.E = V & K. E=$\frac{1}{2}mv^2$)

  $\frac{1}{2}mv^2 = E - V$

  $mv^2 = 2(E - V)$

  $m^2v^2 = 2m\ (E - V)$ --- (6)

- **From (5) & (6)**

  $\nabla^2\psi + (4\pi^2/h^2).\ 2m\ (E - V)\ \psi = 0$

  $\nabla^2\psi + (8\pi^2 m/h^2)\ (E - V)\ \psi = 0$ --------------- (7)

- **Since** $\hbar = h/2\pi$ => $4\pi^2/h = 1/\hbar^2$

  $\nabla^2\psi + (2m/\hbar^2)\ (E - V)\psi = 0$ -------------- (8)

This equation is known as the **Time independent Schrodinger wave equation**.

<u>**NOTE:**</u> For a free particle, V = 0

=> $\nabla^2\psi + (2mE/\hbar^2)\ \psi = 0$ ----------- (9)

This is known as the **Time independent Schrodinger wave equation for free particles**.