1.3.LAWS OF BLACKBODY RADIATION

The following are the various laws of blackbody radiation and these are used in measuring or describing black body radiation in different contexts.

• Stefan-Boltzmann Law:

Introduction:

The Stefan-Boltzmann Law is an empirical relationship obtained by Stefan(1879) and derived theoretically by Boltzmann(1884). It connects the intensity of radiation to the temperature.

- ***Statement*:** The total radiation emitted from a blackbody at temperature T is directly proportional to the fourth power of the absolute temperature of the body.

$$E \propto T^4$$

$$E = \sigma T^4$$

where: σ is called Stefan's constant & $\sigma = 5.67 \times 10^{-8}\ W/m^2 K^4$

• Wien's Law:

Introduction: Wien's Law was developed by German physicist Wilhelm Wien which describes the wavelength at which the emission of a black body spectrum is maximum

- This is more accurate at higher frequencies (shorter wavelengths) in describing the blackbody radiation spectrum and provides a good approximation in this region.

***statement*:** The blackbody radiation curve for different temperatures will peak at different wavelengths and "The maximum (peak) wavelength is inversely proportional to the absolute temperature"

$$\lambda m \propto 1/T$$

$$\lambda m T = constant\ (or)$$

$$\lambda m T = 2.898 \times 10^{-3}\ mK$$

• Rayleigh-Jeans Law:

Introduction: By noticing the failure of Wien's formula on longer wavelengths, Rayleigh & Jeans developed a theory for the spectral distribution of a blackbody by the application of electrodynamics & statistical mechanics.

- Rayleigh-Jeans Law is more accurate at lower frequencies (longer wavelengths) and provides a good approximation in this region.

***statement**:* "The energy density emitted by a blackbody is directly proportional to the temperature & inversely proportional to the wavelength raised to fourth power"

$E_\lambda d_\lambda \propto T/\lambda^4$

Where: $E_\lambda d_\lambda$ is energy density.

• Planck's Radiation Law

Introduction: In 1900, Max Planck introduced the revolutionary concept of radiation known as the quantum theory of radiation.

Assumptions (Hypothesis) made for Planck's radiation law:

- A blackbody radiator contains simple harmonic oscillators of possible frequencies & these oscillators cannot emit or absorb energy continuously.
- The emission or absorption of energy takes place in discrete amounts, i.e., energy of oscillators is quantized.
- The energy of an atomic oscillator of frequency can have only certain values like 0hv, $1h\nu$, $2h\nu$, $3h\nu$, ... $nh\nu$. This is an integral multiple of small units of energy called Quanta or photon.
- In general, for any oscillator of frequency v, the possible values of energy are given by

$E_n = nh\nu$

where: n → +ve integer

ν → frequency

h → Planck's const= 6.625×10^{-34} J.S.

Statement: Planck's radiation law states that "Light travels in the form of small discrete packets of energy called Quanta."

Derivation:

- According to the Maxwell-Boltzmann speed distribution law, the number of oscillators in energy state $En = nh\nu$-------(1), at temperature T *is* given by

$$N_n = N_0 e^{-En/kT}$$

$$N_n = N_0\, e^{-nhv/kT} \text{ -------------- } (2)$$

- **Let** $e^{-hv/kT} = x$

$$N_n = N_0\, \alpha\, x^n \text{ ----------} (3)$$

For $n=0$ *then* $N_0 = N\, x^0 = N_0$

For $n=1$ *then* $N_1 = N_0\, x^1$

For $n=2$ *then* $N_2 = N_0 X^2$

For $n=3$ *then* $N_3 = N_0\, X^3$

For $n=n$ *then* $N_n = N_0 X^n$

- **Let** $N_0, N_1, N_2, N_3, \ldots\ldots N_n$ be the total number of oscillators, then the *total **n**umber of oscillators in the enclosure*

$$N = N_0 + N_1 + N_2 + \ldots\ldots N_n \text{ ---------------------------------- } (4)$$

- ***Now sub***. $N_0, N_1, N_2, N_3, \ldots\ldots N_n$ values in (4)

$$N = N_0 + N_0 x^1 + N_0 x^2 + N_0 x^3 + \ldots\ldots\ldots N_0 x^n \text{ ----------} (5)$$

$$N = N_0\, [1 + x + x2 + x3 + \ldots + x^n]$$

- As per Binomial theorem: $1 + x^1 + x^2 + x^3 + \ldots\ldots\ldots x^n = 1/(1-x)$

$$N = N_0/(1-x) \text{ -----------------} (6)$$

- ***The total energy of oscillators in the enclosure***

$$E = NEn$$

- **From (4) & (1)**

$$E = (N_0 + N_1 + N_2 + N_3 \ldots\ldots Nn)\, nhv \text{ --------------} (7)$$

$$E = N_0 nhv + N_0 nhv + N_0 nhv + N_0 nhv + \ldots + N_n\, nhv$$

$$E = N_0 x^1 1hv + N_0\, x^2\, 2hv + N_0\, x^3\, 3hv + \ldots + N_n\, x^n\, nhv$$

$$E = N_0 hv\, [1 + 2x^1 + 3x^2 + 4x^3 + \ldots\ldots nx^{n-1}]$$

- **As per** Binomial theorem: $1 + 2x^1 + 3x^2 + 4x^3 + \ldots\ldots nx^{n-1} = 1/(1-x)^2$

- **Therefore,** $E = N_0\, xhv\, .1/(1-x)^2$

$$E = N_0 x h\nu / (1-x)^2 \quad \text{- --------------------(8)}$$

- ***The average energy of oscillators in the enclosure***

$$E_{avg} = E/N$$

$$E_{avg} = \{\cancel{N_0}\, xh\nu / (1-x)^2\} / \{\cancel{N_0} / \cancel{(1-X)}\}$$

$$E_{avg} = xh\nu/(1-x) \text{ --------------------------}(9)$$

→**We know**, $x = e^{-h\nu/kT}$

$$E_{avg} = \{\cancel{e^{-h\nu/kT}}.h\nu\} / \{\cancel{e^{-h\nu kT}}\,(1/e^{-h\nu kT} - 1)\}$$

$$E_{avg} = h\nu/(e^{h\nu kT} - 1) \text{ --------------------}(10)$$

- **And** $\nu = c\lambda$

$$E_{avg} = hc/\lambda \,/\, (e^{h\nu/kT} - 1) \text{ --------------------------------}(11)$$

- ***The number of oscillations per unit volume in the frequency range λ & $\lambda + d\lambda$ is determined by*** $dN = \pi/\lambda^4\, d\lambda$ ------------------------*(12)*
- ***But energy density can be obtained by multiplying the number of oscillators per unit volume with the average energy of oscillators***

$$E\lambda d\lambda = E_{avg \times} dN$$

$$E\lambda d\lambda = \{hc/\lambda[1/e^{hc/\lambda kT} - 1]\}8\pi/\lambda^4 d\lambda$$

$$E\lambda d\lambda = 8\pi hc/\lambda 5[1/e^{hc/\lambda kT} - 1]\, d\lambda \quad \text{-------------}(13)$$

(Or)

$$E_\lambda = 8\pi hc/\lambda^5[1/e^{hc/\lambda kT} - 1] \text{ --------------------}14)$$

- ***This is Planck's Radiation Law in terms of wavelengths***

Where: $h = 6.625 \times 10^{-34}$ Js

$c = 3 \times 10^{-8}$ m/s

$k = 1.38x10^{-23}$ J/K (Boltzmann constant)