1.12.FERMI-DIRAC DISTRIBUTION

Introduction:

In 1926,Enrico Fermi and Paul Dirac both came up with Fermi-Dirac distribution which is a key part of quantum statistics.

- In quantum theory, different electrons occupy different energy levels at 0K and obey Pauli exclusion principle.

Explanation:

- As the electrons receive energy, they are excited to higher levels which are unoccupied at 0K.
- The occupation of e's obeys Fermi-Dirac distribution law and the electrons, which obey Fermi-Dirac distribution Law are called Fermions.
- It is easy to find out the e's distribution by using the Fermi-Dirac distribution function.
- The Fermi-Dirac Distribution function at temperature 'T' is given by

$$f(E) = 1 / (e^{(E-Ef)/KT} + 1 \text{ ------------ } (1)$$

- **where:** $E_f \rightarrow$ Fermi energy

  $f(E) \rightarrow$ probability of state of energy

  $E \rightarrow$ Energy
- **And** $E = h^2n^2/8ma^2$-----------------(2)

Filling of Energy levels at T=0K & T>OK:

f(E) at T=0K and Energy level (E) lying:

(i) below Ef $\rightarrow$ **$E<E_f$**

(ii) at $E_f \rightarrow$ **$E=E_f$**

(iii) above Ef $\rightarrow$ **E>Ef**

(i) If $E<E_f$: $E-E_f$ is -ve (: $E<E_f \rightarrow E_f - E \rightarrow -(E-Ef)$)

- $f(E) = 1 / e^{-(E-Ef)/KT)} + 1$

$= 1 / (e^{-\infty} + 1)$

$= 1 / (1/e^{\infty} + 1)$ (since $e^{-\infty} = 1/e^{\infty} = 0$)

$= 1 / (0+1)$

$= 1$

$f(E) = 1$ --------------------(3)

- $f(E) = 1 \rightarrow$ shows that all the energy levels below E_f are filled (occupied) by electrons.

(ii) If $E>E_f$: $E-E_f$ is +ve

- $f(E) = 1 / (e^{(E-E_f)/KT} + 1)$

  $= 1 / (e^{\infty} + 1)$

  $= 1 / (\infty + 1)$ (since $e^{\infty} = \infty$)

  $= 1 / \infty$

  $f(E) = 0$ ------------------ (5)

- $f(E) = 0 \rightarrow$ Shows that all the energy levels above E_f are empty (vacant).

(iii) If E = Ef : E - Ef = 0

- $f(E) = 1 / (e^0 + 1)$ (since $E - E_f = 0$)

  $f(E)$ = Indeterminable -------------(6)

- $f(E) \rightarrow$ Shows that undetermined value ranging between o to 1

f(E) at T > 0K and E = Ef $\rightarrow$ E - Ef = 0

- $f(E) = 1 / (e^{(E-Ef)/KT} + 1)$

  $= 1 / (e^0 + 1)$

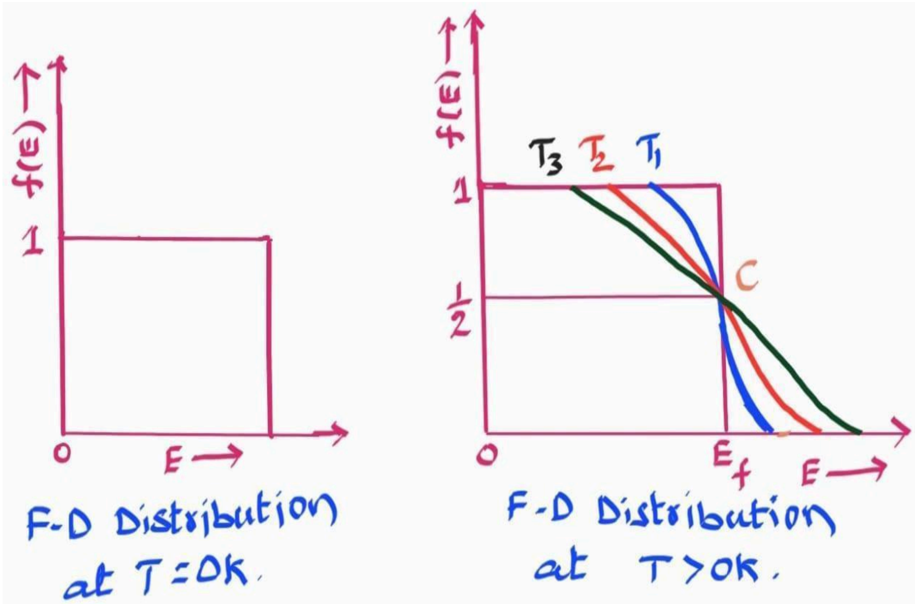
  $= 1 / (1 + 1)$

  $f(E) = 1/2$ ----------------(7)

- $f(E) = 1/2 \rightarrow$ Shows that Fermi level is the state at which the probability of electron occupation is 1/2 (or) 50% at any temperature above 0K.

- The Fermi-Dirac distribution function at three different temperatures T1, T2 & T3

  where T3 > T2 > T1 are given below.



- It is evident that the distribution curves for various temperatures all pass through a common point 'c' where the probability value is 0.5, or 50%."
- This is due to the fact that the probability at point 'c' remains constant at 0.5 (or 50%) for any temperature exceeding 0 Kelvin.

NOTE: $e^0 = 1$; $e^\infty = \infty$; $1/\infty = 0$

(iii) ZONE BAND THEORY OF SOLIDS:

- Even though quantum theory of free electrons is successful in explaining various properties such as heat capacity, thermal conductivity, electrical conductivity, magnetic susceptibility, etc.
- However, this model becomes helpless in explaining several other properties, such as why some materials are conductors and some are insulators, etc.
- In 1928, Bloch introduced the Band theory.
- According to this theory, Free electrons move in a periodic potential provided by the +ve ion core (Lattice).
- This band theory gives complete information about the study of electrons.
- This theory is based on the wave nature of e's and e's exhibit wave character as they move between atoms in a solid.