1.16.EFFECTIVE MASS OF ELECTRON (m*)

Definition: The mass of the electron, when it is accelerated in a periodic potential with the help of an electric field is called the effective mass of the electron.

- Actually, the mass of an electron is varied when it is placed in a periodic potential and this varying mass is an effective mass of the electron. So, it depends on electron location.
- The effective mass is denoted by m* and it is -ve (or) +ve (or) infinite.
- It is given by $m^* = \hbar^2/(d^2E/dk^2)$

Derivation:

- Let us consider the free electron as a wave packet, then the group velocity (Vg) corresponding to angular velocity of a particle is

  $V_g = d\omega/dk$ -------------- (1)

  where: k → wave vector

  ω → Angular velocity of electron

- **Since ω = 2πv** → dω = 2π dv

  $V_g = 2\pi\, dv/dk$ ---------- --- (2)

- **Since E = hv** ⇒ v = E/h ⇒ dv=dE/h

  $V_g = 2\pi/h\ dE/dk$ ---------- (3)

- **But acceleration** of an electron,

  $a = dv_g/dt$

  $= 2\pi/h.\ d/dt\ (dE/dk)$

  $= 2\pi/h.\ d^2E/dk.1/dt$

  $a = 1/\hbar\ d^2E/dk^2\ dk/dt$ ------------------ (4)

- We have, wave propagation vector

  $k = 2\pi/\lambda$

$k = 2\pi/(h/p) = 2\pi p/h$ ($\because \lambda = h/p$)

$k = p/(h/2\pi) \Rightarrow k = p/\hbar \Rightarrow p = \hbar k$ & $dp/dt = \hbar\, dk/dt$

- **Since** $dp/dt = F$

  $F = \hbar\, dk/dt$ --------------- (5)

- **But** $F = m^*a$ ----------- (6)
- **From (4) & (6)**: $F = m^*\, 1/\hbar\, d^2E/dk^2\, dk/dt$ --- -------7)
- **And rom (5) & (7)**: $m^*\, 1/\hbar\, d^2E/dk^2\, dk/dt = \hbar\, dk/dt$

  $m^* = \hbar^2/(d^2E/dk^2)$ ------------ (8)

- This is the equation for effective mass of electrons.

<u>Special Cases:</u>

- If d^2E/dk^2 is positive, then effective mass is positive.
- If d^2E/dk^2 is negative, then effective mass is negative.
- If $d^2E/dk^2 = 0$ then effective mass becomes infinity (∞).