1.7.BORN'S INTERPRETATION OF THE WAVE FUNCTION

Introduction:

In 1926,Max Born proposed his interpretation of the wave function, "stating that the square of its amplitude (or modulus) is proportional to the probability density of finding a particle at a given location".

- In the context of matter waves, a physical quantity that changes periodically is referred to as wave function.
- It is denoted by $\psi(x,t)$, which should be finite, continuous and single valued for all x (position) and t (time).
- A wave function may be used to describe the probability of finding electrons within matter waves.
- Probability interpretation of wave function was given by Max Born in 1926.
- According to Born, the square of the magnitude of the wave function $|\psi|^2$ evaluated in a particular region represents the probability of finding the particles in that region.
- Probability (P) of finding the particle in an infinitesimal volume (dv)$\propto$ $|\psi (x, y, z)|^2$ dxdydz at time t

$$\therefore P \propto |\psi (x, y, z)|^2 dv$$

(or)

$$P \propto |\psi|^2 dv$$

- Since particle is certainly somewhere in a space $\Rightarrow P = 1$

Hence, $\int|\psi|^2 dv = 1$ in the interval $-\infty$ to $+\infty$

- Generally, the wave function ψ is complex. But probability must be a real quantity.

$\therefore$ To make probability real, ψ must be multiplied by its complex conjugate ψ^*.

Thus: $\int \psi\psi^* dv = 1$ in the interval $-\infty$ to $+\infty$

Here: $|\psi|^2 = \psi\psi^*$ and ($dv = dx\, dy\, dz$)

Significance:

- Even Though ψ has no physical significance,

→ $|\psi|^2$ gives the probability of finding the atomic particle in a particular region.

→ $\psi = \psi_0 e^{-i\omega t}$ gives particle behavior.

→ Normalization condition shows $\int |\psi|^2 \, dv = 1$. in the interval $-\infty$ to $+\infty$ to be satisfied by wave function

Conditions to be satisfied by wave function:

- An acceptable wave function

  (i)ψ must be normalized:

  Normalization condition shows $\int |\psi|^2 \, dv = 1$. in the interval $-\infty$ to $+\infty$ to be satisfied by wave function.

  (ii)ψ must be finite:

  The wave function must be finite for all values of x, y, z. If ψ is infinite, it would imply an infinitely large probability of finding the particle at that point.

  (iii)ψ must be single valued:

  Any physical quantity can have only one value at a point.

  (iv)ψ must be continuous:

  Since ψ is related to a physical quantity, its space derivatives $\partial\psi/\partial x$, $\partial\psi/\partial y$ & $\partial\psi/\partial z$ must be continuous at any point.

- The wave function satisfying the above mathematical conditions is called a well-behaved wave function.