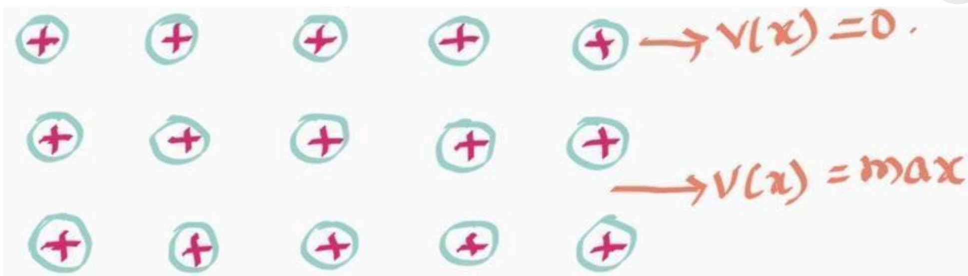
1.13.BLOCH'S THEOREM

Introduction:

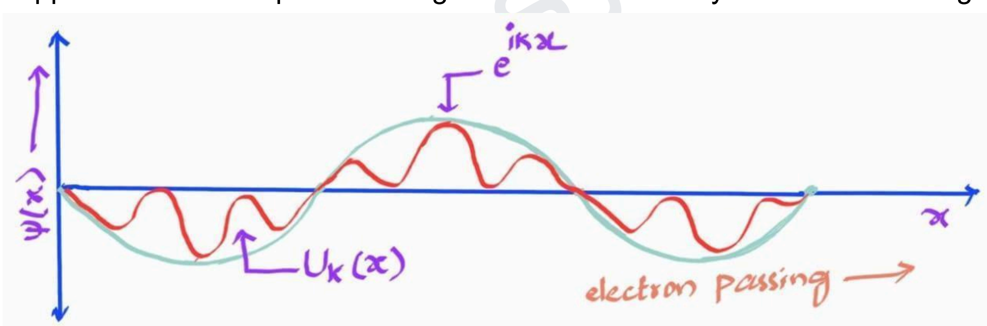
In 1928, Felix Bloch introduced a fundamental concept in condensed matter physics.

Statement: Bloch stated that "the free electrons move in a periodic potential rather than constant potential in between the ion cores in the crystal lattice."

- When an electron moves through the +ve ion cores, it experiences periodic potential.
- The potential is minimum [V(x)=0] at the +ve ion cores and maximum in between two +ve ion cores.



- The solution to the Schrodinger wave equation in a periodic potential can be expressed as plane waves modulated by periodic functions.
- Suppose an electron passes along x-direction in 1-D crystal as shown in figure.



- The periodic potential of moving electron along x-direction is

$$V(x) = V(x+a) \text{ ------------- } (1)$$

- The Schrodinger wave equation for the moving electron along x-direction is

$$\partial^2\psi/\partial x^2 + 8\pi^2 m[E-V(x)]/h^2 = 0 \text{ ----------- } (2)$$

- Bloch has shown the solution for the Schrodinger wave equation (2) is

$$\psi(x) = e^{ikx}\, U_k(x) \text{ ----------- } (3)$$

- But the periodic function with a periodicity 'a' is given by

$$U_k(x) = U_k(x+a) \text{ ------------ } (4)$$

- According to statement the Bloch's function with a periodicity "a" (3) is

$$\psi(x+a) = e^{ik(x+a)}\, U_k(x+a)$$

$= e^{ikx}. e^{ika} U_k(x+a)$

$\psi(x+a) = e^{ika} . e^{ikx} U_k(x+a)$ ------------- (5)

- **from (4**) => $\psi(x+a) = e^{ika}. e^{ikx} U_k(x)$
- **from (3)** => $\psi(x+a) = e^{ika} \psi(x)$ --------------- (6)

(or)

$\psi(x+a) = Q \psi(x)$ ---------------- (7)

where $Q = e^{ika}$ →Phase factor

$U_k(x)$ → Periodic function with period (x+a)

- This equation Eq. (7) is known as Bloch condition.

By taking the complex conjugate of (7) we understand that an electron is not localized around any particular atom and the probability of finding the electron is the same throughout the crystal.

Hence in Eq. (7), $e^{ika} \psi(x)= Q \psi(x)= 1$.